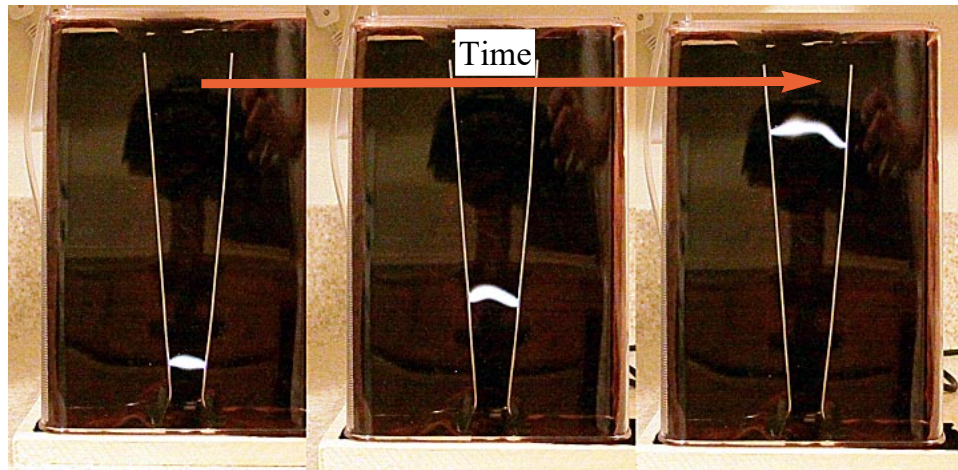


# Lecture 1 - And then there was Light!

## A Puzzle...

Many fields of physics are often interrelated. As shown in the example below, electrical sparks can form between two highly wires, but when they do the spark will always move upwards. This is not an electric effect. Can you explain what is happening?



## Solution

This phenomenon is more intimately tied to the behavior of gases than to electricity. The spark between the two wires (essentially the electrons of the air particles) is hot and therefore rise, and the path of least resistance for the spark follows this rising air. This suggests that if you turn the contraption on its side, it should no longer work, as shown in this nice [explanatory video](#). □

## Introduction

Welcome to Ph 1b, Section 6! Congratulations on making it through one term of Caltech. That is not an insignificant accomplishment, so take a second to enjoy it...now onwards and upwards!

## TA's Information

Name: Tal Einav

Website: <http://www.its.caltech.edu/~teinav>

Office Hours: Tuesday from 4-5pm in Broad 156

## Mathematics

This course will assume that you know multivariable calculus. If that worries you, take [Math 13](#) or read this [Math Bootcamp](#) to get yourself speed.

## Text

During the Special Relativity portion of this class, I will be following David Morin's incredible text *Special Relativity: For the Enthusiastic Beginner*. Once we get into electricity we will follow the class text (David Morin and Edward Purcell's *Electricity and Magnetism*). The moral of the story is: if you read all of David Morin's books, you will master this course.

## Mathematica

*Mathematica* is free for Caltech students, and I highly recommend you download and invest the time to [learn this language](#). My lecture notes will be posted both as *Mathematica* files and as PDFs, but only the former are interactive.

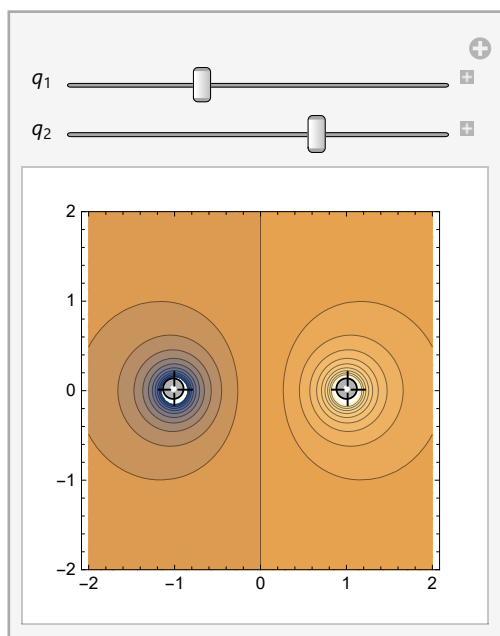
I also use *Mathematica* notation in my notes. For example,  $\arctan\left(\frac{A_y}{A_x}\right)$  will be written as `ArcTan` $\left[\frac{A_y}{A_x}\right]$  using hard brackets “[...]” instead of parenthesis “(...)” and using camel case (i.e. capitalizing the first letter of each word (in this case the “A” and “T”). Similarly, I will write  $\cos(\pi)$  as `Cos` $[\pi]$ . Although this notation looks a bit strange, it has the advantage that it can be plugged directly into *Mathematica* and evaluated,

`Cos`  $[\pi]$

– 1

One big advantage is that you will see neat *Mathematica* applications, especially once we get into electrodynamics. For example, the following models the potential of two electrostatic particles (we will learn about this later in the course). If you are viewing this in *Mathematica* you can change the charge of each particle by moving the sliders  $q_1$  and  $q_2$  at the top, and you can also drag the particles around using the mouse!

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## A Word to the Wise

### Asking Questions

On average, how many questions do you ask in one class period? If your answer is less than 3, you are not asking enough questions. *If you don't ask me a question during recitation sessions, then I am wasting my time!* In such a case, I may as well record myself and not come at all.

Help make the sections much more dynamic by asking for clarification when you do not understand something. And if the pace is fast and you simply need some time to digest what is happening, simply raise your hand and ask for some time.

### Effective Learning

Effective learning is active. Don't listen to lectures or read the book passively. Go through derivations yourself (or try coming up with your own!). Think about other scenarios and see if you can understand them.

Learning also requires you to fail. The best learners allow themselves to make many mistakes along their journey. No fear, no shame - be proud every time that you try!

## Special Relativity - Kinematics

After the special relativity part of this course, you will be able to:

- Understand and apply the postulates of special relativity
- Solve problems from any inertial reference frame
- Judge whether special relativity has any true paradoxes

In this part of the course, we will learn that everything that we have learned in Classical Mechanics is wrong. Or, perhaps, "incomplete" is a better term for it. For it turns out that Newtonian physics is a limiting case of the more correct relativistic theory. It would be silly (to put it mildly) to use relativity to solve a problem involving the length of a baseball trajectory. But in problems involving large speeds, or in problems where a high degree of accuracy is required, you must use the relativistic theory.

Note: You shouldn't feel too bad about having spent so much time learning about a theory that is simply the limiting case of another theory, because you're now going to do it again. Relativity is also the limiting case of yet another theory (quantum field theory). And likewise, you shouldn't feel too bad about spending so much time on relativity, because quantum field theory is also the limiting case of yet another theory (string theory). And likewise...well, you get the idea. It just might actually be turtles all the way down.

Relativity is broken up into two main topics: special relativity (which does not deal with gravity) and general relativity (which does). In both fields, light is one of the key players.

### Light

The speed of light *exactly* equals

$$c = 2.99792458 \times 10^8 \frac{\text{m}}{\text{s}} \quad (1)$$

but in this class we will approximate it as

$$c \approx 3 \times 10^8 \frac{\text{m}}{\text{s}} \quad (2)$$

This is one of the few numbers that you will see so frequently that it will be forever ingrained in your heart.

Because the speed of light is much greater than the speed of everyday objects, most of the consequences of relativity are not noticeable in our daily lives. If we lived in a world similar to ours, with the only difference being that the speed of light were 100 mph, then the consequences of relativity would be ubiquitous. We wouldn't think twice about time dilations, length contractions, and so on.

An interesting note: Although we will not use it in this class, a common - and somewhat surprising - convention is to set  $c = 1$  (note that in this convention,  $c$  is unitless!). Why is such a convention useful and how can it be correct?

As we will see in this course, the speed of light  $c$  shows up all over the place in both special relativity and electrodynamics. Setting  $c = 1$  simplifies these formulas down. To transform from this convention back into SI units, you consider the units of your answer, the units of the desired answer, and multiply by whatever power of  $c$  is necessary to equate the two. For example, if you were using this convention and found that an object has an energy 1 kg, then in SI units this would imply an energy  $c^2$  kilograms, which now indeed has the right units. Note that while setting  $c = 1$  simplifies many equations, it prevents you from using unit analysis to check your work! In this course, we will keep  $c = 3 \times 10^8 \frac{m}{s}$  and stay in SI.

## Complementary Section: Postulates

*(Complementary Sections describe material that was covered in class.)*

The theory of relativity rests upon certain postulates. Recall that an inertial reference frame is a reference frame which is not accelerating or rotating. A reference frame moving with a *constant* velocity relative to an inertial reference frame is also an inertial reference frame.

- The speed of light has the same value in any inertial reference frame.

To understand what this means, consider a train moving along the ground at constant velocity (that is, it is not accelerating; this is the definition of an inertial frame). Someone on the train shines a light from one point on the train to another. Let the speed of the light with respect to the train be  $c$ . Then the above postulate says that a person on the ground also sees the light move at speed  $c$ .

This is a rather bizarre statement. It does not hold for everyday objects. If a baseball is thrown on a train, then the speed of the baseball is of course different in different frames. The observer on the ground must add the velocity of the train and the velocity of the ball (with respect to the train) to obtain the velocity of the ball with respect to the ground (although, as we will find out, this is not exactly true; we will derive the exact velocity addition formula later).

The truth of the speed-of-light postulate cannot be demonstrated from first principles. In the end, we must rely on experiment. And indeed, all the consequences of the speed-of-light postulate have been verified countless times during this century. In particular, the consequences are being verified continually each day in high-energy particle accelerators, where elementary particles reach speeds very close to  $c$ .

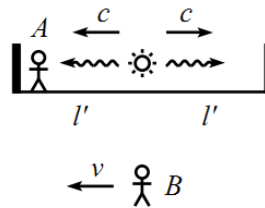
- All inertial frames are “equivalent”.

This postulate basically says that one inertial frame is no better than any another. There is no preferred reference frame. That is, it makes no sense to say that something is moving; it only makes sense to say that one thing is moving with respect to another. This is where the “relativity” in special relativity comes from. There is no absolute frame; the motion of any frame is only defined relative to other frames.

## Loss of Simultaneity

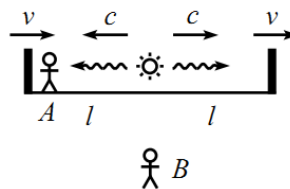
Consider the following setup. In  $A$ 's reference frame, a light source is placed midway between two receivers, a distance  $l'$  from each. The light source emits a flash. From  $A$ 's point of view, the light hits the two receivers at the same time,  $\frac{l'}{c}$  seconds after the flash.

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Now consider another observer,  $B$ , who travels to the left at speed  $v$ . In  $B$ 's reference frame, the situation looks markedly different

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The receivers (along with everything else in  $A$ 's frame) are moving to the right at speed  $v$ , and the light is traveling in both directions at speed  $c$  with respect to  $B$  (not with respect to the light source, as measured in  $B$ 's frame; this is where the speed-of-light postulate comes into play). Therefore, in  $B$ 's frame the light will hit the left receiver at time  $t_l = \frac{l}{c+v}$  and right receiver at time  $t_r = \frac{l}{c-v}$ . These are not equal if  $v \neq 0$ , except for in the special case of  $l = 0$  (in which case the two events occur simultaneously in all reference frames).

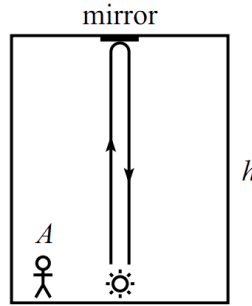
Note that the two distances  $l'$  and  $l$  will be different due to length contraction, but that does not matter here; all that is important is that a simultaneous event in  $A$ 's reference frame does not occur simultaneously in  $B$ 's reference frame. The moral of this exercise is that it makes no sense to say that one event happens at the same time as another, unless you state which frame you're talking about. Simultaneity depends on the frame in which the observations are made.

Let us contrast this with the Newtonian physics that we have used throughout classical mechanics. If instead of light we were talking about two baseballs which are thrown at velocity  $v_b$  in  $A$ 's reference frame, then  $B$  would see these baseballs moving at speeds  $v_b - v$  to the left and  $v_b + v$  to the right. The relative speeds between the baseballs and the left and right receivers would therefore be  $(v_b - v) + v = v_b$  and  $(v_b + v) - v = v_b$ . These are equal, so  $B$  would see these two baseballs hit the receiver at the same time, as we well know from everyday experiences.

## Complementary Section: Time Dilation

Let there be a light source on the floor of the train and a mirror on the ceiling, which is a height  $h$  above the floor. Let observer  $A$  be on the train, and observer  $B$  be on the ground. The speed of the train is  $v$  (with respect to the ground). A flash of light is emitted. The light travels up to the mirror and then back down to the source. In  $A$ 's frame, the train is at rest, with the path of light being vertically upwards and then vertically downwards.

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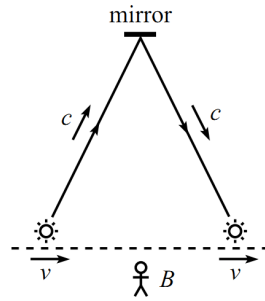


It takes the light a time  $\frac{h}{c}$  to reach the ceiling and then a time  $\frac{h}{c}$  to return to the source. The round-trip time is therefore

$$t_A = \frac{2h}{c} \tag{3}$$

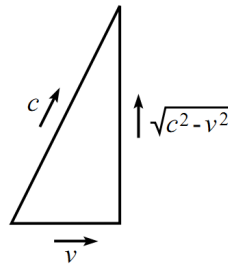
In  $B$ 's frame, the train moves at speed  $v$  with the path of the light moving at diagonals due to the train's motion.

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The crucial fact to remember is that the speed of light in  $B$ 's frame is still  $c$ . This means that the light travels along its diagonally upward path at speed  $c$ . (The vertical component of the speed is not  $c$ , as would be the case if light behaved like a baseball.) Since the horizontal component of the light's velocity is  $v$ , the vertical component must be  $\sqrt{c^2 - v^2}$ .

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The time it takes to reach the mirror is therefore  $\frac{h}{\sqrt{c^2 - v^2}}$  so the round-trip time is

$$t_B = \frac{2h}{\sqrt{c^2 - v^2}} \tag{4}$$

Therefore we find that the time dilation formula

$$t_B = \gamma t_A \tag{5}$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} \tag{6}$$

is a convenient factor that is ubiquitous factor in relativity. Note that  $\gamma \geq 1$  with equality iff (if and only if)  $v = 0$ . Therefore, if the train is moving,  $t_B > t_A$ .

For concreteness, suppose the train was moving with velocity  $v = \frac{3}{5}c$  so that  $\gamma = \frac{5}{4}$ . Then we may say the following. If  $A$  is standing next to the light source, and if  $B$  is standing on the ground, and if  $A$  claps his hands at  $t_A = 4$  second intervals, then  $B$  will observe claps at  $t_B = 5$  second intervals (after having subtracted off the time for the light to travel to her eye, of course). This is true because both  $A$  and  $B$  must agree on the number of round-trips the light beam takes between claps. Assuming, for convenience, that a round-trip takes one second in  $A$ 's frame, the four round-trips between claps will take five seconds in  $B$ 's frame from the above equation  $t_B = \gamma t_A$ . And if we have a train that does not contain one of our special clocks, that's no matter. We could have built one if we wanted to, so the same results concerning the claps must hold. Therefore,  $B$  will observe  $A$  moving strangely slowly.  $B$  will observe  $A$ 's heart-beat beating slowly; his blinks will be a bit lethargic; and his sips of coffee will be slow enough to suggest that he needs another cup.

The time dilation result  $t_B = \gamma t_A$  is a bit strange, no doubt, but there doesn't seem to be anything downright incorrect about it until we look at the situation from  $A$ 's point of view.  $A$  sees  $B$  flying by at a speed  $v$  in the other direction. The ground is no more fundamental than a train, so the same reasoning applies. The time dilation factor,  $\gamma$ , doesn't depend on the sign of  $v$ , so  $A$  sees the same time dilation factor that  $B$  sees. That is,  $A$  sees  $B$ 's clock running slow. But how can this be? Are we claiming that  $A$ 's clock is slower than  $B$ 's, and also that  $B$ 's clock is slower than  $A$ 's?

Well...yes and no. It is true that in  $A$ 's frame,  $B$ 's clock runs slow while in  $B$ 's frame  $A$ 's clock runs slow. But this is not a contradiction. Observers  $A$  and  $B$  are using different coordinates to measure time. The times measured in each of their frames are quite different things. The seemingly contradictory time-dilation result is really no stranger than having two people run away from each other into the distance, and having them both say that the other person looks smaller.

You may still find it unsettling that  $A$  sees  $B$ 's clock run slow and that  $B$  sees  $A$ 's clock run slow in their own reference frames. You may be thinking, "This is a contradiction. It is essentially the same as saying, 'I have two apples on a table. The left one is bigger than the right one, and the right one is bigger than the left one.'" But no, we are not comparing apples and apples. We are comparing apples and oranges. A more correct analogy would be the following. An apple and an orange sit on a table. The apple says to the orange, "You are a much uglier apple than I am," and the orange says to the apple, "You are a much uglier orange than I am."

## Understanding Time Dilation

So we cannot directly compare what the notion of "time" means in two different reference frames. But we can make definitive statements about the time that any event takes in two different reference frames. In the example above, the event was the light leaving the source, bouncing off the mirror, and then returning to the source. How long does this event take in frame  $A$  and frame  $B$ ?

But now you find yourself in a conundrum: you may analyze the situation in  $A$ 's reference frame where  $B$  is moving or instead in  $B$ 's reference frame where  $A$  is moving. The former seems to indicate that  $t_A = \gamma t_B$  while the latter indicates that  $t_B = \gamma t_A$ . However, only one of these statements can be true (otherwise we must have  $t_A = t_B = 0$ ). How do you determine which one? You simply have to note one thing...

**Our assumption that  $A$  is at rest on the train was critical in the above derivation.** Therefore, only if in frame  $A$  did two events occur in one location can you use the time dilation formula  $t_B = \gamma t_A$  above.

Why was the assumption that  $A$  is at rest critical in the above derivation? If  $A$  is moving with respect to the train, then the above equation  $t_B = \gamma t_A$  does not hold, because we cannot say that both  $A$  and  $B$  must agree on the

number of round-trips the light beam takes between claps, because of the problem of simultaneity. More precisely, if  $A$  is at rest on the train right next to the light source, then there are no issues with simultaneity because the distance between  $A$  and  $B$  is zero (assuming the train is tiny and the people are point masses, with  $B$  practically touching the train as it passes). And if  $A$  is at rest a fixed distance from the source, then consider a person  $A'$  at rest on the train right next to the source. The distance  $L$  between  $A$  and  $A'$  is non-zero, so there will be a "Head Start" effect (discussed in the next Section), so from the loss of simultaneity,  $B$  sees their clocks read different times. But the difference is constant, so  $B$  sees the clock  $A$  tick at the same rate as the clock  $A'$ . Equivalently, if  $A$  was holding a light clock, that clock would have the same speed  $v$  (and thus yield the same  $\gamma$  factor) as the first clock.

However, if  $A$  was moving with respect to the train, then we have a problem. If  $A'$  is again at rest next to the source, then the distance  $L$  between  $A$  and  $A'$  is changing, so  $B$  can't use the reasoning in the previous paragraph to conclude that  $A$ 's and  $A'$ 's clocks tick at the same rate. And in fact they do not, because as above, we can build another light clock and have  $A$  hold it. In this case,  $A$ 's speed is what goes into the  $\gamma$  factor in  $\gamma = 1 / \left(1 - \frac{v^2}{c^2}\right)^{1/2}$ , but this is different from  $A'$ 's speed (which is the speed of the train).

Lastly, you may ask: what if two events happened in one location in frame  $A$  (so that we can use  $t_B = \gamma t_A$ ), and these same two events also occurred in the same location in frame  $B$  (so that we can use  $t_A = \gamma t_B$ )? There are only two ways for this to happen:

1. Either the two frames are not moving relative to one another (in which case they are both the same frame,  $v = 0$ , and there will not be any time dilation (or, equivalently, there will be time dilation with the factor  $\gamma = 1$ )).
2. The two events occur at the same location *and at the same time*. In this case, these two events will occur at the same location and at the same time in all reference frames (so once again, the time dilation formula holds).